*Journal of Economic Theory and Econometrics*, Vol. 30, No. 2, Jun. 2019, 1–19

# On the Micro-Level Irrelevance of Habit Formation in Complete Markets

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**Abstract** This paper characterizes conditions for external habit formation to render habit-driven equilibrium outcomes observationally equivalent to habit-free equilibrium outcomes in multi-agent complete markets. The uniformity of the ratios of habit levels to aggregate consumptions across consumption goods leads to the irrelevance of habit formation to excess demand and asset prices. Thus, external habit formation may have no tangible effect on excess demand and the equity premium if the cross-sectional habitual variations are not substantial. The micro-level analysis of habit formation produces useful information about the habitual effect on the individual welfare and the existence of competitive equilibrium which cannot be addressed in a representative agent framework.

**Keywords** External habit, observational irrelevance, competitive equilibrium, complete markets, relative risk aversion

**JEL Classification** G12, D53, D51, D15

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*Received November 20, 2018, Revised March 10, 2019, Accepted March 15, 2019*

# 1. INTRODUCTION

Asset pricing models with habit formation provide an insight into the cyclical behavior of aggregate stock prices, which is hard to capture in habit-free pricing models. Campbell and Cochrane (1999) build a model with external habit formation, wherein the representative agent has a utility function with constant risk aversion regarding the difference between consumption and habit. They attempt to explain the asset pricing implications of habit formation to the equity premium and risk-free rate puzzles. The habit formation restricts the ability of agents to accumulate for smoothing consumption over time, and thus, to substitute consumption intertemporally, especially when they fall in a low-income state. The relative risk aversion in the habit model is inversely related to the ratio of consumption beyond the habit level to the whole consumption. This implies that relative risk aversion must increase as the surplus-consumption ratio declines. Especially, agents become extremely risk averse when they make consumption choices near the habit level. In this case, they demand a high risk premium for investing in risky assets. Such a cyclical behavior of relative risk aversion can contribute to variations in risk premia over business cycles.

The paper provides a necessary and sufficient condition for the external habit formation to render habit-driven equilibrium outcomes observationally equivalent to habit-free equilibrium outcomes in multi-agent complete markets. The irrelevance of habit formation to excess demand and asset prices holds if and only if the ratios of habit formation to the aggregate consumption are identical across all goods. Thus, external habit formation may have no tangible effect on excess demand and equity premium if the cross-sectional habitual variations are not substantial. The results of the paper are obtained from transforming the economy with external habit into a habit-free economy with income shocks. Agents are assumed to have the same constant relative risk aversion for consumption that exceeds the habit level, and the same probabilistic beliefs on future events.

The micro-level analysis of habit formation is differentiated from the macro-level or representative-agent model in several respects. First, the representative-agent model for habit formation cannot take into account the effect of the cross-sectional distributions of habit formation on the individual welfare. Other things being equal, agents with higher habit level have higher marginal utility of income which leads to lower Pareto weight in the linear social welfare program. If two agents have the same preferences and endowments except for the habitual level, the agent with higher habit would be more affected by income taxes or income shocks whether they are positive or negative. Second, the micro-level approach allows us to understand how habit formation affects the individual allocation of wealth between riskless and risky assets. The presence of habit makes relative risk aversion vary over time. The time-varying behavior of relative risk aversion due to habit formation affects the attractiveness of risky assets relative to riskless assets. Finally, the macro-level habit model does not explain the effect of the distributions of habit formation on the existence of competitive equilibrium. Competitive equilibrium exists in the macro-level habit model as far as the aggregate endowment is greater than the aggregate habit level for each consumption good. However, as illustrated later, competitive equilibrium may fail to exist in the multi-agent (or micro-level) habit model if individual habitual consumption is large relative to the individual endowment of the consumption good.

The literature on external habit formation is briefly reviewed. Campbell and Cochrane (1999) introduce the external habit model in which utility depends on the difference between consumption and external habit to explain equity premium and the cyclical behavior of stock prices and return volatility. The difference model for habit formation is discussed in Ljungqvist and Uhlig (2000, 2015). Ljungqvist and Uhlig (2000) examine the macroeconomic implications of tax policies aimed at correcting the welfare distortion of consumption externality, which arises from the behavior of "keeping up with the Joneses" and "catching up with the Joneses." In contrast to Campbell and Cochrane (1999) wherein an infinitesimal destruction of the endowment leads to a welfare loss, Ljungqvist and Uhlig (2015) demonstrate that discrete endowment perturbations can lead to a welfare gain through the life-time reduction in consumption externality of habit formation in the framework of Campbell and Cochrane (1999). Brunnermeier and Nagel (2008) provide a test whether difference habit formation affects stock market participation and the allocation of liquid wealth between riskless and risky assets. They document that changes in liquid wealth have a significant effect on the participation decision but essentially play no role in explaining changes in household asset allocation. In the habit model, consumption affects welfare relative to the habit formation. The role of relative consumption is also emphasized in the literature on positional goods such as Frank (1985, 2005). Consumption externality with positional goods, however, differs from the externality of habit formation in that negative externality is imposed on agents who fail to attain the full standard of the positional goods. For instance, a worker would be uneasy from falling behind his peers in the hierarchy of his company.

This paper is in the same spirit as Gali (1994) who studies the effect of consumption externalities of keeping up with the Joneses in the framework of Abel (1990) where utility is determined by the ratio between consumption and habit formation. Gali (1994) characterizes risk aversion for an externality-free economy which makes it share the same equilibrium asset prices as an economy with consumption externalities. In contrast, the current paper with difference-form habit attempts to find conditions on the magnitude of habit relative to aggregate consumptions which will guarantee the irrelevance of habit formation. Interestingly, Chetty and Szeidl (2016) show that the habit model of Campbell and Cochrane (1999) can be explained from the perspective of adjustment costs for the dynamics of aggregate consumption. Consumption commitments provide a reference point for aggregate consumption which acts like habit formation.

## 2. THE MODEL

The paper starts with a static general equilibrium model with finite goods. Multiperiod complete markets with a single good are discussed in Section 5. It is well known that a dynamic complete-market economy can be converted into a static general equilibrium model. Specifically, the paper studies the effect of external habit formation on equilibrium outcomes in an exchange economy $\mathcal{E}^H = \{(X_i', u_i', e_i') : i \in I\}$ with $\ell$ goods where

a) $I = \{1, \ldots, m\}$ is the set of consumers, and $J = \{1, \ldots, \ell\}$ the set of consumption goods.

b) $X_i'$ is the consumption set of agent $i$ in $\mathbb{R}_+^\ell$,

c) $u_i'$ is the utility function of agent $i$ over $X_i'$,

d) $e_i' = (e_{i,1}', \ldots, e_{i,\ell}') \in \mathbb{R}_+^\ell$ is agent $i$'s initial endowment of consumption goods.

Following the external habit model pioneered by Campbell and Cochrane (1999), agent $i$ is assumed to have the utility function in $\mathbb{R}_+^\ell$

$$u_i'(x_i) = \sum_{j=1}^{\ell} \phi_j \frac{(x_{i,j} - \bar{x}_{i,j})^{1-\rho}}{1-\rho},$$

where $\bar{x}_{i,j} \geq 0$ is the level of habit for the $j$th good, $\rho > 0$ measures the constant relative risk aversion (CRRA) of agent $i$ for consumption that exceeds the habit level, and $\phi_j > 0$ is a positive parameter. In the presence of the habit parameters $\bar{x}_i = (\bar{x}_{i,1}, \ldots, \bar{x}_{i,\ell})$, agent $i$ is required to choose a consumption in the set

$$X_i' \equiv \{x_i \in \mathbb{R}_+^\ell : x_i \geq \bar{x}_i\}.^1$$

For a price $p \in \mathbb{R}_{++}^\ell$, agent $i$ makes an optimal choice which maximizes his utility in the budget set $B_i(p, e_i') \equiv \{x_i \in X_i' : p \cdot x_i \leq p \cdot e_i'\}$. An equilibrium for the economy $\mathcal{E}^H$ is defined as follows.

**Definition 1.** A price-allocation pair $(p, (x_i')_{i \in I})$ is an equilibrium of the economy $\mathcal{E}^H$ if it satisfies the following conditions:

i) each $x_i'$ maximizes $u_i(x_i)$ over all $x_i$'s in $B_i(p, e_i')$, and

ii) it holds that $\sum_{i \in I}(x_i' - e_i') = 0$.

For each $i$, the following terms arise from translating $X_i'$ and $e_i'$ by $\bar{x}_i$, respectively:

$$X_i \equiv X_i' - \bar{x}_i = \mathbb{R}_+^\ell \quad \text{and} \quad e_i \equiv e_i' - \bar{x}_i.$$

[1] For vectors $x, y$ in $\mathbb{R}^\ell$, $x \geq y$ if and only if every element in $x - y$ is nonnegative, $x > y$ if and only if $x \geq y$ and $x \neq y$, and $x \gg y$ if and only if every element of $x - y$ is positive.

The habit $\bar{x}_i$ can be interpreted as a negative endowment shock to agent $i$. Then the economy $\mathcal{E}^H$ with external habit is transformed into a habit-free economy $\mathcal{E} = \{(X_i, u_i, e_i) : i \in I\}$ with endowment shocks where $u_i$ is a utility function in $X_i$ defined by

$$u_i(x_i) = \sum_{j=1}^{\ell} \phi_j \frac{x_{i,j}^{1-\rho}}{1-\rho}.$$

The habit-free economy $\mathcal{E}$ shares the same equilibrium outcomes with the economy $\mathcal{E}^H$ except that optimal consumptions differ by the habit levels. This result is formalized as following.

**Proposition 1.** A price-allocation pair $(p, (x_i')_{i \in I})$ is an equilibrium of the economy $\mathcal{E}^H$ if and only if $(p, (x_i)_{i \in I})$ where $x_i = x_i' - \bar{x}_i$ for each $i \in I$ is an equilibrium of the habit-free economy $\mathcal{E}$.

Proposition 1 allows us to use the economy $\mathcal{E}$ to discuss the relevance of habit formation to equilibrium outcomes instead of $\mathcal{E}^H$.

It is worth noting that agent $i$ in $\mathcal{E}$ has constant relative risk aversion $\rho$ in the habit-free economy $\mathcal{E}$ but the relative risk aversion varies with habit level in $\mathcal{E}^H$ as following.

$$\frac{\rho\, x_{i,j}}{x_{i,j} - \bar{x}_{i,j}}$$

As remarked below, for some $j \in J$, $e_{i,j}$ will be negative when the habit formation of agent $i$ for good $j$ exceeds the initial endowment $e'_{i,j}$ for the economy $\mathcal{E}^H$, i.e., $\bar{x}_{i,j} > e'_{i,j}$. In this case, $e_i$ need not lie in the consumption set $X_i = \mathbb{R}^{\ell}_{+}$. This poses a very delicate issue for the existence of equilibrium in general. As shown later, however, the existential issue is clearly addressed in the current setting which allows for the closed-form solution for equilibrium outcomes.

Now let $\mathcal{E}^N$ denote the special version of the economy $\mathcal{E}^H$ in which $\bar{x}_i$ is set to 0 for all $i \in I$. The economy $\mathcal{E}^N$ in which agents have no habit formation differs from the economy $\mathcal{E}$, the habit-free equivalent of $\mathcal{E}^H$. In particular, the economy $\mathcal{E}$ shares the same equilibrium price with $\mathcal{E}^H$ but $\mathcal{E}^N$ need not. To examine the relationship between equilibrium outcomes of $\mathcal{E}$ and $\mathcal{E}^N$, we define the notion of observational equivalence.

**Definition 2.** Let $(p,x)$ and $(p',x')$ be an equilibrium for $\mathcal{E}$ and $\mathcal{E}^N$, respectively. The two economies are observationally equivalent if it holds that

i) $p = \lambda p'$ for some $\lambda > 0$, and

ii) $x_i - e_i = x'_i - e'_i$ for all $i \in I$.

The first condition requires both $\mathcal{E}$ and $\mathcal{E}^N$ to have the same equilibrium price up to the price normalization. The second condition represents the equivalence between net trades in $\mathcal{E}$ and $\mathcal{E}^N$ which are observable in markets.

**Remark 1.** The initial endowment $e_i$ may not lie in the consumption set $X_i$. For instance, let's take a two-good economy where agent $i$ possesses $e'_i = (5,1)$ and $\bar{x}_i = (2,2)$. Then $e_i = (3,-1)$, the initial endowment with the habit level deducted, is not in $X_i = \mathbb{R}^2_+$. The initial endowment $e_i$ does not allow agent $i$ to subsist. Thus, the self-subsistence condition breaks down which is assumed for the existence of equilibrium in the classical literature. Fortunately, the existential issue is neatly resolved in the current framework which allows for closed-form equilibrium outcomes.

**Remark 2.** Both economies $\mathcal{E}$ and $\mathcal{E}^N$ have a unique equilibrium if they are observationally equivalent. To see this, let $(p,x)$ and $(q,y)$ be two equilibria for $\mathcal{E}$, and $(p',x')$ and $(q',y')$ be two equilibria for $\mathcal{E}^N$. Comparisons are made between $(p,x)$ and $(p',x')$ and then between $(p',x')$ and $(q,y)$. By the observational equivalence, it holds that $p = \lambda p'$ for some $\lambda > 0$ and $x_i - e_i = x'_i - e'_i$ for all $i \in I$, and $p' = \lambda' q$ for some $\lambda' > 0$ and $x'_i - e'_i = y_i - e_i$ for all $i \in I$. The result yields $p = \lambda\lambda' q$ and $x_i = y_i$ for all $i \in I$. Thus, $\mathcal{E}$ has a unique equilibrium with normalized prices. By the symmetric arguments, $\mathcal{E}^N$ has a unique equilibrium with normalized prices as well.

## 3. THE CLOSED-FORM EQUILIBRIUM OUTCOMES

Agents in $\mathcal{E}^H$ are heterogeneous with respect to both the initial endowments and the habit formation. As discussed in the previous section, $\mathcal{E}^H$ is transformed into the habit-free economy $\mathcal{E}$ in which agents are heterogeneous with respect to the initial endowments. By Proposition 1, $\mathcal{E}^H$ has an equilibrium if and only if $\mathcal{E}$ does. This section presents conditions under which both $\mathcal{E}$ and $\mathcal{E}^N$ have a unique equilibrium. The following provides the unique existence of equilibrium in the economy $\mathcal{E}$.

**Theorem 1.** Suppose that $\mathcal{E}$ satisfies the following conditions

1) $e_{0,j} \equiv \sum_{i\in I} e_{i,j} > 0$ for each $j \in J$, and
2) $\sum_{j\in J} e_{0,j}^{-\rho} e_{i,j} \phi_j > 0$ for each $i \in I$.

Then $\mathcal{E}$ has a unique equilibrium $(p,x)$ where for each $i \in I$ and $j \in J$,

$$p_j = \phi_j e_{0,j}^{-\rho} \quad \text{and} \quad x_{i,j} = e_{0,j} \sum_{k\in J} \left( \frac{e_{0,k}^{-\rho+1}\phi_k}{\sum_{k\in J} e_{0,k}^{-\rho+1}\phi_k} \right) \frac{e_{i,k}}{e_{0,k}}. \tag{1}$$

PROOF : For a price $p \gg 0$, the first-order condition for maximizing $u_i(x_i)$ subject to the budget constraint $p\cdot(x_i - e_i) \leq 0$ yields

$$x_{i,j} = \left(\frac{\phi_j}{\lambda_i p_j}\right)^{1/\rho}, \tag{2}$$

where $\lambda_i > 0$ is the Lagrangian multiplier. We set $\mu_i = (1/\lambda_i)^{1/\rho}$. By putting (2) into the budget constraint, we obtain

$$\mu_i = \frac{\sum_{j\in J} p_j e_{i,j}}{\sum_{j\in J} p_j \left(\frac{\phi_j}{p_j}\right)^{1/\rho}}. \tag{3}$$

It follows from the market clearing condition that for each $j \in J$,

$$e_{0,j} = \sum_{i\in I} x_{i,j} = \left(\frac{\phi_j}{p_j}\right)^{1/\rho} \sum_{i\in I} \mu_i = \left(\frac{\phi_j}{p_j}\right)^{1/\rho} \frac{\sum_{j\in J} p_j e_{0,j}}{\sum_{j\in J} p_j \left(\frac{\phi_j}{p_j}\right)^{1/\rho}}. \tag{4}$$

For each $j \in J$, it gives

$$\frac{\sum_{j\in J} p_j e_{0,j}}{\sum_{j\in J} p_j \left(\frac{\phi_j}{p_j}\right)^{1/\rho}} = e_{0,j} \left(\frac{\phi_j}{p_j}\right)^{-1/\rho}.$$

Since the term in the left-hand side is constant, it holds that

$$e_{0,1}\Big(\frac{\phi_1}{p_1}\Big)^{-1/\rho} = \cdots = e_{0,\ell}\Big(\frac{\phi_\ell}{p_\ell}\Big)^{-1/\rho}.$$

We normalize prices by setting

$$e_{0,1}\Big(\frac{\phi_1}{p_1}\Big)^{-1/\rho} = 1.$$

Then the economy has a unique equilibrium price $p$ with $p_j = \phi_j e_{0,j}^{-\rho}$. By putting $p_j = \phi_j e_{0,j}^{-\rho}$ into (3), we obtain

$$\mu_i = \frac{\sum_{j\in J} e_{0,j}^{-\rho} e_{i,j}\phi_j}{\sum_{j\in J} e_{0,j}^{-\rho+1}\phi_j} > 0. \quad \text{(by condition 2))} \tag{5}$$

The optimal consumption $x_i$ in (1) is obtained from putting $p$ and $\mu_i$ into (2). $\square$

The following corollary is immediate from the fact that $\mathcal{E}^N$ is identical to $\mathcal{E}$ when each $e'_i$ replaces $e_i$.

**Corrolary 1.** Suppose that $\mathcal{E}^N$ satisfies the following conditions

1) $e'_{0,j} \equiv \sum_{i\in I} e'_{i,j} > 0$ for each $j \in J$, and
2) $\sum_{j\in J}(e'_{0,j})^{-\rho} e'_{i,j}\phi_j > 0$ for each $i \in I$.

Then $\mathcal{E}^N$ has a unique equilibrium $(p', x')$ where for each $i \in I$ and $j \in J$,

$$p'_j = (\phi_j e'_{0,j})^{-\rho} \quad \text{and} \quad x'_{i,j} = e'_{0,j}\sum_{k\in J}\Big(\frac{(e'_{0,k})^{-\rho+1}\phi'_k}{\sum_{k\in J}(e'_{0,k})^{-\rho+1}\phi'_k}\Big)\frac{e'_{i,k}}{e'_{0,k}}. \tag{6}$$

The following example illustrates that the failure of Condition 2 of Theorem 1 leads to the nonexistence of equilibrium in the economy $\mathcal{E}^H$.

**Example 1:** Suppose that there are two agents in the economy $\mathcal{E}^H$ with $\ell = 2$ and $\rho = 2$ where they have the initial endowments $e'_1 = (3,0), e'_2 = (0,3)$. The habit levels and the utility parameters are given by

$$\bar{x}_1 = \bar{x}_2 = (1,1), \phi_1 = \frac{1}{4}, \phi_2 = \frac{3}{4}.$$

In other words, agent $i$ has a utility function

$$u_i(a,b) = -\frac{1}{4}(a-1)^{-1} - \frac{3}{4}(b-1)^{-1}.$$

Then Condition 2) of Theorem 1 is violated:

$$e_{0,1}^{-2} e_{1,1}\phi_1 + e_{0,2}^{-2} e_{1,2}\phi_2 = -\frac{1}{4}.$$

This implies the Lagrangian multiplier for agent 1 in (5) is negative, which is impossible in equilibrium. Thus, the economy has no equilibrium. □

## 4. THE IRRELEVANCE OF EXTERNAL HABIT FORMATION

This section presents a necessary and sufficient condition for habit formation to be irrelevant to equilibrium prices and excess demand. To do this, for each $(i,j) \in I \times J$ let $\varepsilon_{i,j} \geq 0$ denote the proportion of agent $i$'s habit level to the aggregate endowment $e'_{0,j}$, i.e.,

$$\varepsilon_{i,j} = \frac{\bar{x}_{i,j}}{e'_{0,j}}.$$

The relation leads to

$$e_{i,j} = e'_{i,j} - \bar{x}_{i,j} = e'_{i,j} - \varepsilon_{i,j} e'_{0,j}. \tag{7}$$

The following shows that $\mathcal{E}$ and $\mathcal{E}^N$ are observationally equivalent if and only if each agent displays the same degree of external habit relative to aggregate consumption across all consumption goods.

**Theorem 2.** Suppose that the two conditions 1) and 2) in Theorem 1 hold. The two economies $\mathcal{E}$ and $\mathcal{E}^N$ are observationally equivalent if and only if for each $i \in I$, $\varepsilon_{i,j}$'s are identical across $J$, i.e., there exists $\varepsilon_i \geq 0$ such that

$$\varepsilon_i = \varepsilon_{i,1} = \varepsilon_{i,2} = \cdots = \varepsilon_{i,\ell}. \tag{8}$$

PROOF : Let $(p,x)$ and $(p',x')$ denote the unique equilibrium of $\mathcal{E}$ and $\mathcal{E}^N$, respectively. Suppose that $\varepsilon_{i,j}$'s satisfy (8). By summing up $e_{i,j} = e'_{i,j} - \varepsilon_i e'_{0,j}$ over

$i \in I$, we obtain

$$e_{0,j} = e'_{0,j} - \Big(\sum\nolimits_{i\in I} \varepsilon_i\Big) e'_{0,j} = e'_{0,j}\Big(1 - \sum\nolimits_{i\in I} \varepsilon_i\Big).$$

The result leads to

$$\frac{e'_{0,1}}{e_{0,1}} = \frac{e'_{0,2}}{e_{0,2}} = \cdots = \frac{e'_{0,\ell}}{e_{0,\ell}}. \tag{9}$$

Then the optimal consumptions in (1) yield

$$\begin{aligned}
x_{i,j} - e_{i,j} &= e_{0,j}\Bigg(\sum\nolimits_{k\in J}\Big(\frac{e_{0,k}^{-\rho+1}\phi_k}{\sum_{k\in J} e_{0,k}^{-\rho+1}\phi_k}\Big)\Big(\frac{e_{i,k}}{e_{0,k}} - \frac{e_{i,j}}{e_{0,j}}\Big)\Bigg)\\
&= e'_{0,j}\Bigg(\sum\nolimits_{k\in J}\Big(\frac{(e'_{0,k})^{-\rho+1}\phi_k}{\sum_{k\in J}(e'_{0,k})^{-\rho+1}\phi_k}\Big)\Big(\frac{e'_{i,k} - \varepsilon_i e'_{0,k}}{e'_{0,k}} - \frac{e'_{i,j} - \varepsilon_i e'_{0,j}}{e'_{0,j}}\Big)\Bigg)\\
&= e'_{0,j}\sum\nolimits_{k\in J}\Big(\frac{(e'_{0,k})^{-\rho+1}\phi_k}{\sum_{k\in J}(e'_{0,k})^{-\rho+1}\phi_k}\Big)\Big(\frac{e'_{i,k}}{e'_{0,k}} - \frac{e'_{i,j}}{e'_{0,j}}\Big)\\
&= x'_{i,j} - e'_{i,j} \quad \text{(by (6))},
\end{aligned}$$

where the second equality comes from (9). As each $i$ has the same excess demand in both $\mathcal{E}$ and $\mathcal{E}^N$, they have the same equilibrium price up to price normalization, i.e., $p = \lambda p'$ for some $\lambda > 0$.

Conversely, suppose that $\mathcal{E}$ and $\mathcal{E}^N$ are observationally equivalent, i.e., $p = \lambda p'$ for some $\lambda > 0$ and

$$x'_{i,j} - e'_{i,j} = x_{i,j} - e_{i,j}. \tag{10}$$

By putting the equilibrium prices in (1) and (6) into the relation $p = \lambda p'$, we obtain (9). This result along with the allocations $x$ in (1) and $x'$ (6) leads to

$$\begin{aligned}
x'_{i,j} - x_{i,j} =& e'_{0,j}\sum\nolimits_{k\in J}\Big(\frac{(e'_{0,k})^{-\rho+1}\phi_k}{\sum_{k\in J}(e'_{0,k})^{-\rho+1}\phi_k}\Big)\frac{e'_{i,k}}{e'_{0,k}} - e_{0,j}\sum\nolimits_{k\in J}\Big(\frac{e_{0,k}^{-\rho+1}\phi_k}{\sum_{k\in J} e_{0,k}^{-\rho+1}\phi_k}\Big)\frac{e_{i,k}}{e_{0,k}}\\
&= e_{0,j}\sum\nolimits_{k\in J}\Big(\frac{e_{0,k}^{-\rho+1}\phi_k}{\sum_{k\in J} e_{0,k}^{-\rho+1}\phi_k}\Big)\Big(\frac{e'_{i,k}}{e_{0,k}} - \frac{e_{i,k}}{e_{0,k}}\Big)\\
&= e_{0,j}\sum\nolimits_{k\in J}\Big(\frac{e_{0,k}^{-\rho+1}\phi_k}{\sum_{k\in J} e_{0,k}^{-\rho+1}\phi_k}\Big)\frac{\varepsilon_{i,k} e'_{0,k}}{e_{0,k}}
\end{aligned}$$

By combining the result with (7) and(10), we see that for all $(i,j) \in I \times J$,

$$\sum_{k\in J} \left( \frac{e_{0,k}^{-\rho+1}\phi_k}{\sum_{k\in J} e_{0,k}^{-\rho+1}\phi_k} \right) \frac{\varepsilon_{i,k} e'_{0,k}}{e_{0,k}} = \frac{\varepsilon_{i,j} e'_{0,j}}{e_{0,j}}.$$

As the term in the left-hand side is independent of $j$, it holds that for all $i \in I$,

$$\frac{\varepsilon_{i,1} e'_{0,1}}{e_{0,1}} = \frac{\varepsilon_{i,2} e'_{0,2}}{e_{0,2}} = \cdots = \frac{\varepsilon_{i,\ell} e'_{0,\ell}}{e_{0,\ell}}.$$

The result combined with (9) ensures that for each $i \in I$,

$$\varepsilon_{i,1} = \varepsilon_{i,2} = \cdots = \varepsilon_{i,\ell}.$$

□

## 5. HABIT FORMATION IN MULTI-PERIOD MARKETS

This section makes an application of the results in Sections 3 and 4 to a potentially-complete-market economy with a single good which persists in $(T+1)$ periods, $0,1,\ldots,T$.[2] When the economy arrives in a non-terminal node, it faces uncertainty represented by $S$ events in $\mathcal{S} = \{1,2,\ldots,S\}$. For each $t = 1,\ldots,T$, we set $\mathcal{S}_t = \{0\} \times \mathcal{S}^t$ with $\mathcal{S}_0^0 = \{0\}$. A point in $\mathcal{S}_T$ where 0 is the initial period represents a state, a possible whole path along which uncertainty is resolved up to the terminal time $T$. A point $s^t = (0,s_1,\ldots,s_t) \in \mathcal{S}_t$ is a state (up to time $t$) with $s^0 = 0$ which describes the resolution of uncertainty up to time $t$. Let $\mathcal{E}_d^H$ denote a dynamic complete-market version of $\mathcal{E}^H$ in the following sense. A contingent good in the dynamic economy $\mathcal{E}_d^H$ is indexed by a point in $\mathcal{J}_d \equiv \mathcal{S}_0 \cup \mathcal{S}_1 \cup \cdots \cup \mathcal{S}_T$. To exploit the results in Sections 3 and 4, the total number of contingent goods is set to equal $\ell$, i.e., $\ell = 1 + S^1 + \cdots + S^T$. A contingent good at $s^t$ can match a good in the static economy by a bijective map $L$ between $J = \{1,\ldots,\ell\}$ and $\mathcal{J}_d$, i.e., $L(s^t) = j$ for some $j \in J$. Any allocation in $\mathcal{E}_d^H$ can be mapped into an allocation in $\mathcal{E}^H$ through the map $L$ and vice versa. For the sake of market completeness, it is assumed that there exist sufficiently many assets to provide

[2]The potential completeness of asset markets is explained below.

full insurance against uncertainty in a generic sense.[3] Let $K$ denote a finite set of long-lived assets which are traded during the duration of $\mathcal{E}_d^H$. Let $d_{k,s^t}$ denote the dividends that asset $k \in K$ pays at each $s^t \in \mathcal{S}_t$ with $t = 1, \ldots, T$, and $q_{k,s^t}$ and $\theta_{i,s^t}^k$ denote the price of asset $k$ and agent $i$'s holdings of asset $k$ at each $s^t \in \mathcal{S}_t$ with $t = 0, 1, \ldots, T-1$. The next-period payoffs at state $s^{t-1}$ is summarized into a $K \times S$ matrix

$$\begin{bmatrix} d_{1,(s^{t-1},1)} + q_{1,(s^{t-1},1)} & \cdots & d_{1,(s^{t-1},S)} + q_{1,(s^{t-1},S)} \\ \vdots & \ddots & \vdots \\ d_{K,(s^{t-1},1)} + q_{K,(s^{t-1},1)} & \cdots & d_{K,(s^{t-1},S)} + q_{K,(s^{t-1},S)} \end{bmatrix}$$

Asset markets are potentially complete if each $K \times S$ dividend matrix $[d_{k,(s^{t-1},s)}]$ has rank $S$. Let $\Theta$ denote the space of asset holdings for agent $i$ which consists of a point $\theta_i = (\theta_\sigma^i)_{\sigma \in \mathcal{J}_d'}$ with $\theta_\sigma^i \in \mathbb{R}^K$ where $\mathcal{J}_d' = \mathcal{J}_d \setminus \mathcal{S}_T$. For a price $(p, q)$, a choice $(x_i, \theta_i) \in X_i' \times \Theta$ is budget-feasible if for all $(s^{t-1}, s_t) \in \mathcal{S}_t$, it satisfies

$$x_{i,(s^{t-1},s_t)} - e'_{i,(s^{t-1},s_t)} = (d_{(s^{t-1},s_t)} + q_{(s^{t-1},s_t)}) \cdot \theta_{i,s^{t-1}} - q_{(s^{t-1},s_t)} \cdot \theta_{i,(s^{t-1},s_t)}$$

where $d_{0,s_1} = 0$ for all $s_1 \in \mathcal{S}_0$ and $\theta_{i,(s^{T-1},s_T)} = 0$ for all $s_T \in \mathcal{S}_0$. Let $B_i^d(p, q, e_i')$ denote the budget set for agent $i$, the set which contains all the budget-feasible choices $(x_i, \theta_i)$'s. An equilibrium for the economy $\mathcal{E}_d^H$ is defined as follows.

**Definition 3.** A price-allocation pair $(p, q, (x_i', \theta_i')_{i \in I})$ is an equilibrium of the economy $\mathcal{E}_d^H$ if it satisfies the following conditions:

i) each $(x_i', \theta_i')$ maximizes $u_i(x_i)$ over all $(x_i, \theta_i)$'s in $B_i^d(p, q, e_i')$, and

ii) it holds that $\sum_{i \in I}(x_i' - e_i') = 0$ and $\sum_{i \in I} \theta_i' = 0$.

The growth rate in the initial endowment process $\{e'_{i,s_t}\}$ is assumed to follow a Markov chain: for each $t = 1, \ldots, T$,

$$e'_{i,s^t} = g_{i,s^t} e'_{i,s^{t-1}}, \tag{11}$$

[3]Here real assets such as dividend-paying stocks are long-lived so that their market span is endogenously determined. In this case, the potentially complete asset markets attain equilibrium with Pareto optimal allocations except for exceptional cases. For this matter, see Magill and Quinzii (1996).

where the growth rate $g_{i,s^t}$ takes a value in $\{\bar{g}^i_1, \dots, \bar{g}^i_S\}$. Let $\pi_{s,s'}$ denote the transition probability that the growth rate takes $\bar{g}^i_{s'}$ in the coming period given that it takes $\bar{g}^i_s$ now.

$$\pi_{s,s'} = \Pr\{g_{i,s^t} = \bar{g}^i_{s'} \mid g_{i,s^{t-1}} = \bar{g}^i_s\}.$$

Agent $i$ has a utility

$$E_0\Big\{\sum_{t=0}^{T} \beta^t \frac{(x_{i,s^t} - \bar{x}_{i,s^t})^{1-\rho}}{1-\rho}\Big\},$$

where $E_0$ is the expectation at time 0 with respect to the probability measure determined by $\pi$, $\bar{x}_{i,s^t}$ indicates the level of external habit at state $s^t$, and $\beta \in (0,1)$ is the time discount factor common to all the agents. Agent $i$ has the endowment $e_{i,s^t} = e'_{i,s^t} - \bar{x}_{i,s^t}$ in the economy $\mathcal{E}_d$. Let $e'_{0,s^t}$ denote the aggregate endowment for $\mathcal{E}^H_d$ at state $s^t$.

$$e'_{0,s^t} = \sum_{i \in I} e'_{i,s^t}$$

There exists an equilibrium in the decentralized economy $\mathcal{E}^H_d$ if and only if for all $(i,\sigma) \in I \times \mathcal{J}_d$, it holds that $e_{0,\sigma} > 0$ and

$$E_0\Big\{\sum_{\tau=0}^{T} \beta^\tau (e'_{0,s^\tau} - \bar{x}_{0,s^\tau})^{-\rho}(e'_{i,s^\tau} - \bar{x}_{i,s^\tau})\Big\} > 0. \tag{12}$$

This condition is a dynamic version of Condition 2 of Theorem 1. As illustrated in Example 1, $\mathcal{E}^H_d$ fails to attain equilibrium if (12) is violated.

Let $\mathcal{E}_d$ and $\mathcal{E}^N_d$ denote a counterpart of $\mathcal{E}$ and $\mathcal{E}^N$ in the dynamic context, respectively. Suppose that $\mathcal{E}_d$ has a Pareto optimal equilibrium where agent $i$ makes an optimal consumption $x_i$. Then by nesting (1) in the dynamic setting, $x_i$ is expressed as

$$x_{i,s^t} = e_{0,s^t} E_0\Big\{\sum_{\tau=0}^{T} \beta^\tau \frac{(e_{0,s^\tau})^{-\rho+1}}{E_0\big\{\sum_{\tau=0}^{T} \beta^\tau (e_{0,s^\tau})^{-\rho+1}\big\}} \frac{e_{i,s^\tau}}{e_{0,s^\tau}}\Big\} \quad \text{for all } s^t \in \mathcal{S}_t. \tag{13}$$

At each $s^t \in \mathcal{S}_t$ with $t = 0, 1, \dots, T-1$, asset $k$ has the value

$$q_{k,s^t} = E_t\Big\{\sum_{\tau=t+1}^{T} \beta^{\tau-t} \Big(\frac{e_{0,s^\tau}}{e_{0,s^t}}\Big)^{-\rho} d_{k,s^\tau}\Big\}, \tag{14}$$

where $E_t$ is the expectation conditional on information available at time $t$.

The following provides the notion of observational equivalence between the dynamic economies $\mathcal{E}_d$ and $\mathcal{E}_d^N$.

**Definition 4.** Let $(q,x)$ and $(q',x')$ be an equilibrium for $\mathcal{E}_d$ and $\mathcal{E}_d^N$, respectively.[4] The two economies are observationally equivalent if it holds that

i) $q = \lambda q'$ for some $\lambda > 0$, and

ii) $x_i - e_i = x'_i - e'_i$ for all $i \in I$.

For each $(i,\sigma) \in I \times \mathcal{J}_d$, let $\varepsilon_{i,\sigma} \geq 0$ denote the proportion of agent $i$'s habit level to the aggregate endowment $e'_{0,\sigma}$. Two types of habit formation are considered here. The first case is "keeping up with the Joneses" (Gali (1994)) where the habit level $\bar{x}_{i,s^t}$ is determined by the current aggregate consumption:

$$\bar{x}_{i,s^t} = \varepsilon_{i,s^t} e'_{0,s^t}. \tag{15}$$

The other case is "catching up with the Joneses" (Abel (1990)) where $\bar{x}_{i,s^t}$ is determined by the lagged aggregate consumption[5]:

$$\bar{x}_{i,s^t} = \varepsilon_{i,s^t} e'_{0,s^{t-1}} = \frac{\varepsilon_{i,s^t}}{g_{i,s^t}} e'_{0,s^t}. \tag{16}$$

When agents care about the lagged aggregate consumption, higher endowment growth rate makes habit formation in (16) less sensitive to the current aggregate endowment. It is assumed that $\varepsilon_{i,s^t}$ takes a value in $\{\delta^i_1, \ldots, \delta^i_S\}$. It holds that under the first habit formation,

$$e_{i,s^t} = e'_{i,s^t} - \varepsilon_{i,s^t} e'_{0,s^t} \tag{17}$$

while under the second one,

$$e_{i,s^t} = e'_{i,s^t} - \frac{\varepsilon_{i,s^t}}{g_{i,s^t}} e'_{0,s^t}. \tag{18}$$

[4]Trading strategies are intentionally omitted from equilibrium outcomes for notational simplicity because they have no explicit role in discussing the phenomenon of observational irrelevance.

[5]Following Campbell and Cochrane (1999), the current paper takes habit formation in difference form while it is of ratio form in Abel (1990) and Gali (1994).

The following provides the irrelevance of habit formation in the dynamic context.

**Theorem 3.** Suppose that $\mathcal{E}_d$ and $\mathcal{E}_d^N$ have a Pareto optimal equilibrium.

a) (Keeping up with the Joneses) $\mathcal{E}_d$ and $\mathcal{E}_d^N$ are observationally equivalent if and only if for each $i \in I$, $\varepsilon_{i,s^t}$'s are identical across $\mathcal{J}_d$, i.e.,

$$\varepsilon_{i,s^1} = \cdots = \varepsilon_{i,s^T}. \tag{19}$$

b) (Catching up with the Joneses) $\mathcal{E}_d$ and $\mathcal{E}_d^N$ are observationally equivalent if and only if for each $i \in I$, $\varepsilon_{i,s^t}/g_{i,s^t}$'s are identical across $\mathcal{J}_d$, i.e.,

$$\frac{\varepsilon_{i,s^1}}{g_{i,s^1}} = \cdots = \frac{\varepsilon_{i,s^T}}{g_{i,s^T}}. \tag{20}$$

PROOF : Let $(q,x)$ and $(q',x')$ denote a Pareto optimal equilibrium of $\mathcal{E}_d$ and $\mathcal{E}_d^N$, respectively. The result of Theorem 2 can be applied to the current case by converting $(q,x)$ and $(q',x')$ into an equilibrium of $\mathcal{E}$ and $\mathcal{E}^N$, respectively. Specifically, the relations (19) and (20) is analogously obtained from (8) by comparing the relations (17) and (18) to (7), respectively. The proof is done by matching the indices in $\mathcal{J}_d$ and $J$ through the map $L$ in the proof arguments of Theorem 2. $\square$

Whether habit formation follows the keeping up or catching up with the Joneses rule, the irrelevance of habit formation holds if and only if the ratios of habit levels to the current aggregate consumptions are identical across all the contingencies. For the latter rule, the habit levels are inversely related to the the endowment growth rates.

## 6. COMPARISON WITH MACRO-LEVEL IRRELEVANCE

The equilibrium outcomes of $\mathcal{E}_d$ is easily converted into those of the economy $\mathcal{E}_d^H$. Let $x_i'$ and $q'$ denote the optimal consumption of agent $i$ and the equilibrium asset price in $\mathcal{E}_d^H$. By (13), we see that for each $s^t \in \mathcal{S}_t$,

$$x'_{i,s^t} = \bar{x}_{i,s^t} + (e'_{0,s^t} - \bar{x}_{0,s^t}) E_0 \Big\{ \sum_{\tau=0}^{T} \beta^\tau \frac{(e'_{0,s^\tau} - \bar{x}_{0,s^\tau})^{-\rho+1}}{E_0\big\{ \sum_{\tau=0}^{T} \beta^\tau (e'_{0,s^\tau} - \bar{x}_{0,s^\tau})^{-\rho+1} \big\}} \frac{e'_{i,s^\tau} - \bar{x}_{i,s^\tau}}{e'_{0,s^\tau} - \bar{x}_{0,s^\tau}} \Big\}, \tag{21}$$

where $\bar{x}_{0,s^\tau} \equiv \sum_{i \in I} \bar{x}_{i,s^\tau}$ indicates the aggregate habit level at $s^\tau$. By (14) , the equilibrium price at each $s^t \in \mathcal{S}_t$ with $t = 0, 1, \ldots, T-1$ is given

$$q'_{k,s^t} = E_t \Big\{ \sum_{\tau=t+1}^{T} \beta^{\tau-t} \Big( \frac{e'_{0,s^\tau} - \bar{x}_{0,s^\tau}}{e'_{0,s^t} - \bar{x}_{0,s^t}} \Big)^{-\rho} d_{k,s^\tau} \Big\} \tag{22}$$

The price $q'$ is the equilibrium price of the representative-agent economy where the agent has a utility function

$$E_0 \Big\{ \sum_{t=0}^{T} \beta^t \frac{(x_{0,s^t} - \bar{x}_{0,s^t})^{1-\rho}}{1-\rho} \Big\}.$$

The equilibrium price $q'$ is affected by the aggregate habit formation and thus, independent of the distributions of habit formation across agents.

## 7. CONCLUDING REMARKS

The paper has characterized conditions under which habit formation is observationally irrelevant to equilibrium outcomes such as excess demand for consumption goods and asset prices. Specifically, the habit-driven equilibrium outcomes are observationally equivalent to habit-free equilibrium outcomes if and only if individual habit formation relative to aggregate consumption is identical across consumption goods. The result implies that habit formation observationally matters to excess demand and equity premia when the strength of habit formation relative to the aggregate endowments differs across consumption goods. Thus, it would be hard to detect an empirical evidence about the effect of habit formation on equilibrium outcomes if the relative strength of habit formation is not substantially time-varying.

The current discussion is restricted to complete-market economies where agents have identical constant relative risk aversion and homogeneous beliefs.

Especially, it seeks closed-form solutions for equilibrium outcomes. The closed-form solution approach has limitation in an economy where agents have either distinct relative risk aversion or asset markets are incomplete.[6] It will be a challenging task to study the observational irrelevance of habit formation in a case where equilibrium outcomes admit no closed-form solutions.

[6]It is hard to find explicit forms of equilibrium outcomes in these two cases.

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